Name solutions

EE 311 Final Exam Fall 2009

December 19, 2009

Closed Text and Notes

1) Be sure you have 15 pages.

2) Write only on the question sheets. Show all your work. If you need more room for a particular problem, use the reverse side of the same page.

3) Write neatly, if your writing is illegible then print.

4) The last 6 pages contain equations that may be of use to you.

5) This exam is worth 200 points.

(12 pts) 1.Shown are two co-centric conducting shells. The first conducting shell has inner radius a and outer radius b. The second conducting shell has inner radius c and outer radius d. The outer surface of the outer conductor is tied to ground. The two conducting shells are initially uncharged. A point charge of +Q is placed at the center of the shells. What is the total charge in each of the following regions.

Inner surface of the inner shell -Q

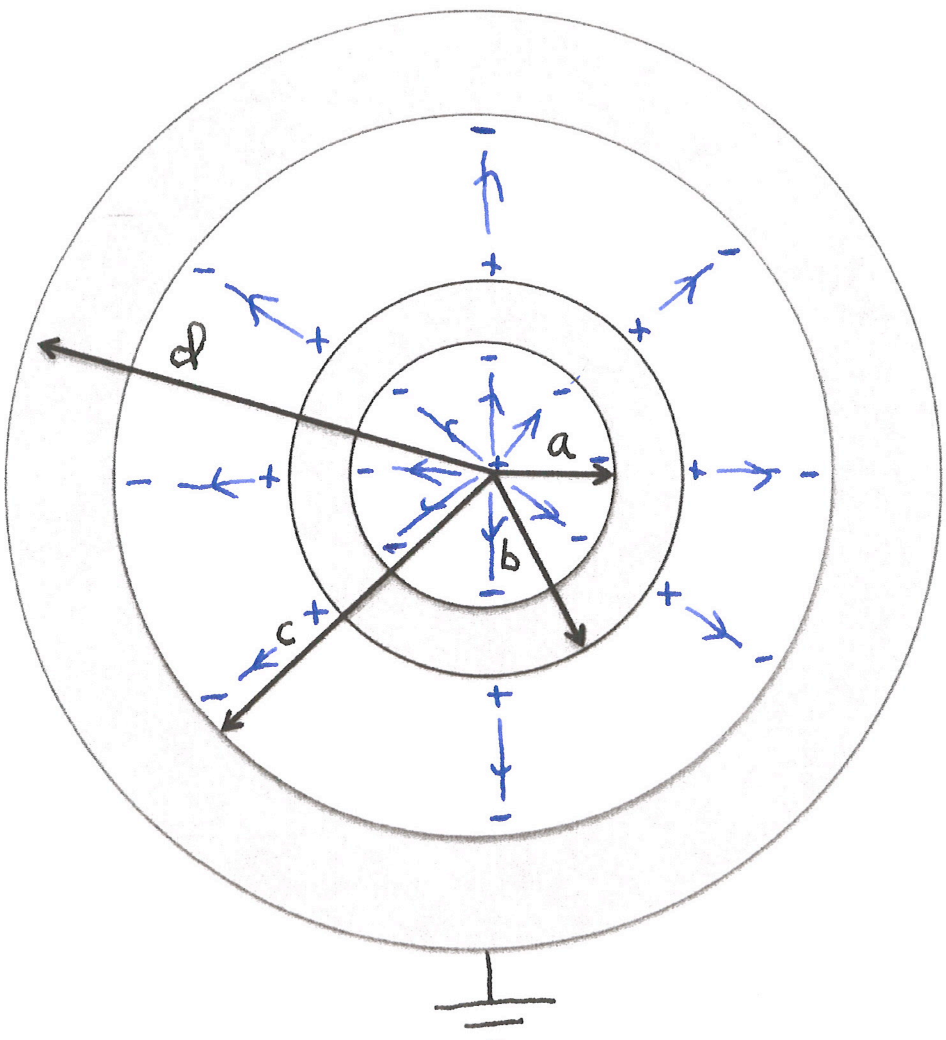
Interior of the inner shell 0

Outer surface of the inner shell +Q

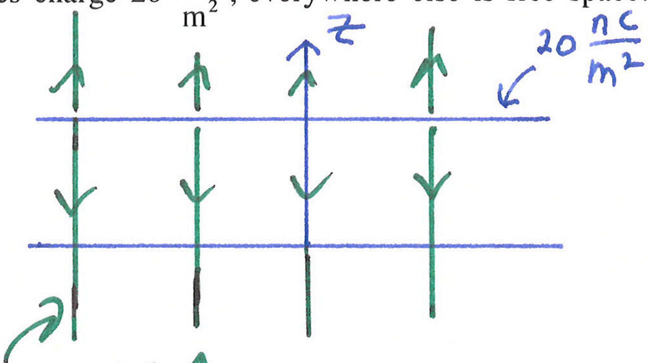
Inner surface of the outer shell -Q

Interior of the outer shell 0

Outer surface of the outer shell 0



(5 pts) 2. The plane z = 10 m carries charge 20 $\frac{\text{nC}}{\text{m}^2}$, everywhere else is free space. The electric field intensity at the origin is,



A) $-10\hat{\mathbf{a}}_z \frac{\text{V}}{\text{m}}$

B) $-18\pi\hat{\mathbf{a}}_z \frac{\text{V}}{\text{m}}$

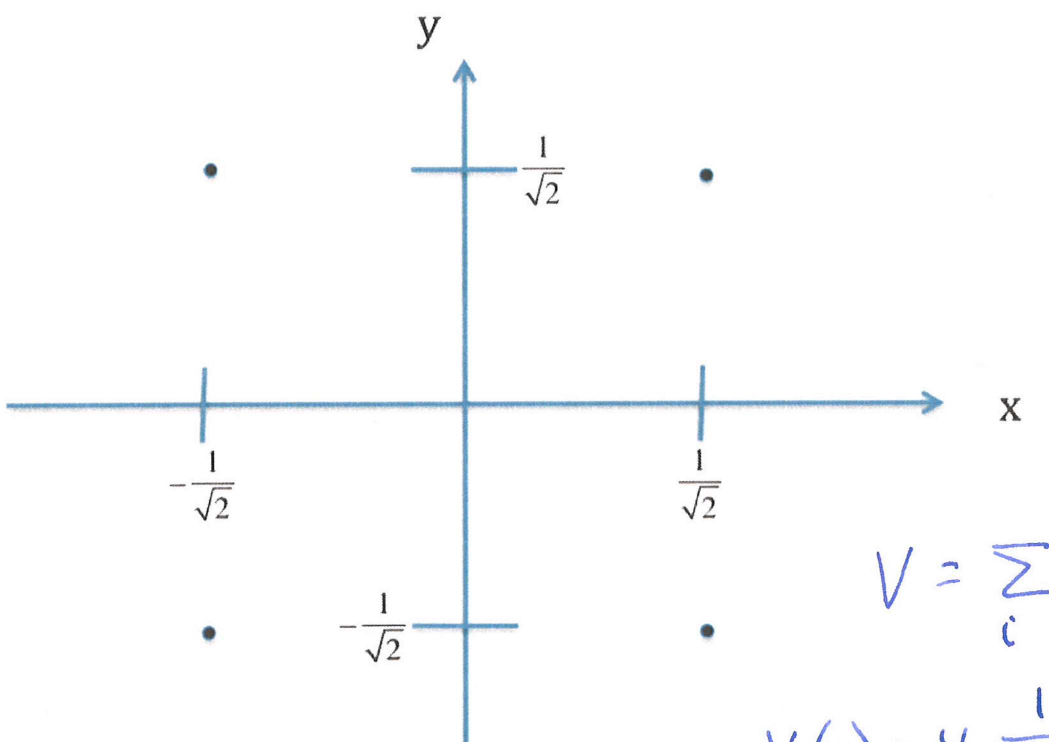
C) $-72\pi\hat{\mathbf{a}}_z \frac{\text{V}}{\text{m}}$

D) $-360\pi\hat{\mathbf{a}}_z \frac{\text{V}}{\text{m}}$

$\vec{D} = -10 \frac{nC}{m^2} \hat{a}_z$

$\vec{E} = \frac{\vec{D}}{\epsilon_0} = -\frac{10^{-8} \frac{C}{m^2}}{(10^{-9}/36\pi) F/m} \hat{a}_z = -360\pi \hat{a}_z \frac{V}{m}$

(5 pts) 3. For the arrangement of charges shown each of value $\frac{10^{-9}}{36}$C, and



With $V(r = \infty) = 0$, the potential at the origin is

A) 0V

B) ∞ V

C) 4 V

D) 1 V

E) none of the above

$$V = \sum_i \frac{Q_i}{4\pi\epsilon_0 r_i}$$

$$V(0) = 4 \frac{1}{4\pi\epsilon_0} \frac{(10^{-9}/36)C}{1m}$$

$$= \frac{1}{\pi(10^{-9}/36\pi)} \frac{(10^{-9}/36)}{1} V$$

$$= 1 V$$

(5 pts) 4. How much energy is stored in an arrangement of two point charges, one of charge $\frac{10^{-4}}{9}$ C at location (0, 1m, 0) and one of charge 10^{-5} C at (0, 2m, 0)?

A) 0 J
B) 0.5 J
C) 1 J
D) 2 J

placing the first charge, the $\frac{10^{-4}}{9}$ C at (0, 1m, 0) requires no work.

The work to place the second charge is $W = \frac{(10^{-4}/9)}{4\pi\epsilon_0} 10^{-5}\,\text{J} = \frac{(10^{-4}/9)\,10^{-5}}{4\pi(10^{-9}/36\pi)}\,\text{J}$

$= 1\,\text{J}$

(5 pts) 6. A parallel-plate capacitor connected to a battery stores twice as much charge with a given dielectric as it doe with air as dielectric. The susceptibility of the dielectric is

A) 0
B) 1
C) 2
D) 3
E) 4

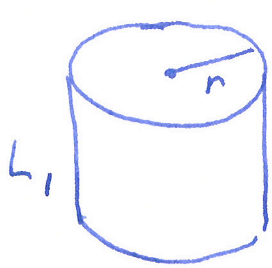
air: $Q = \epsilon_0 A \frac{V}{d}$

dielectric: $2Q = \epsilon_r \epsilon_0 \frac{V}{d}$

$\epsilon_r = 2 = (1+\chi) \Rightarrow \chi = 1$

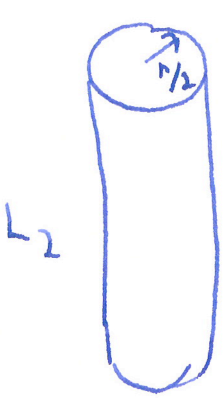
(5 pts) 7. A potential difference V is applied to a mercury column in a cylindrical container. The mercury is now poured into another cylindrical container of half the radius and the same potential difference V is applied across the ends. As a result of this change of space, the resistance is increased

A) 16 times
B) 8 times
C) 4 times
D) 2 times



$\text{Volume} = \pi r^2 L_1$

$R_1 = \rho \frac{L_1}{\pi r^2}$

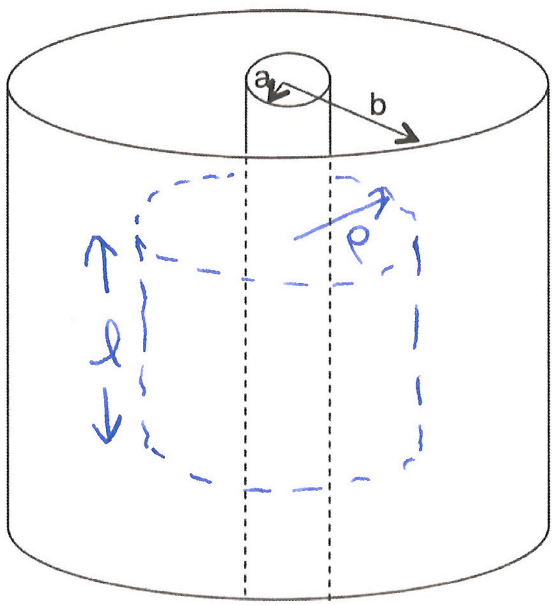


$\text{Volume} = \pi \left(\frac{r}{2}\right)^2 L_2 = \pi r^2 L_1$

$L_2 = 4L_1$

$R_2 = \rho \frac{4L_1}{\pi (r/2)^2} = 16\rho \frac{L_1}{\pi r^2}$

(10 pts) 5. Derive the capacitance per unit length for the coaxial cable shown. The inner conductor has radius a, the outer conductor radius b, and free space is between the conductors.



assume a total charge of $+Q$ on length l of the inner conductor, hence $-Q$ on the inside of the outer conductor.

using Gauss's law on the dashed cylinder

$$\oint \vec{D} \cdot d\vec{s} = Q_{encl}$$

$$D_\rho \, 2\pi \rho l = Q$$

$$\vec{D} = \frac{Q}{2\pi \rho l} \hat{a}_\rho \Rightarrow \vec{E} = \frac{Q}{2\pi \epsilon_0 \rho l} \hat{a}_\rho$$

$$V = -\int_b^a \vec{E} \cdot d\vec{l} = -\int_b^a \frac{Q}{2\pi \epsilon_0 \rho l} \hat{a}_\rho \cdot d\rho \, \hat{a}_\rho$$

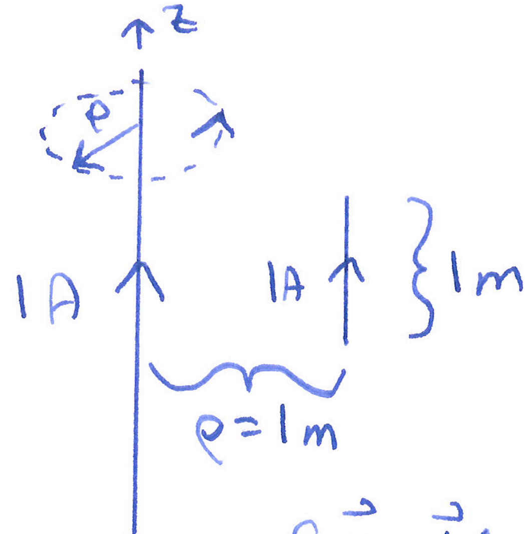
$$= -\frac{Q}{2\pi \epsilon_0 l} \int_b^a \frac{d\rho}{\rho} = -\frac{Q}{2\pi \epsilon_0 l} \ln \rho \Big|_b^a$$

$$= -\frac{Q}{2\pi \epsilon_0 l} \ln\left(\frac{a}{b}\right) = \frac{Q}{2\pi \epsilon_0 l} \ln\left(\frac{b}{a}\right)$$

$$C = \frac{Q}{V} = \frac{Q}{\frac{Q}{2\pi \epsilon_0 l} \ln\left(\frac{b}{a}\right)} = \frac{2\pi \epsilon_0 l}{\ln(b/a)}$$

$$\frac{C}{l} = \frac{2\pi \epsilon_0}{\ln(b/a)}$$

(10 pts) 8. An infinite wire is along the z-axis and carriers a current of 1A in the $\hat{\mathbf{a}}_z$ direction. A second wire of length 1m carries a current of 1 A in the direction $\hat{\mathbf{a}}_z$ direction, is parallel to the first wire and is 1 m away. (You can think of the conduction as a sliding bar of length 1 m on a rail system.) What is the force on the 1 m wire? I want a numerical value including the direction of the force.



$$\vec{F} = +\int I\, d\vec{l} \times \vec{B}$$

so we need to find $\vec{B}$

First find $\vec{H}$ using Ampere's Law around circular path shown

$$\oint \vec{H}\cdot d\vec{l} = I_{enclosed}$$

$$H_\phi\, 2\pi\rho = I \quad\Rightarrow\quad \vec{H} = \frac{I}{2\pi\rho}\,\hat{a}_\phi$$

$$\vec{B} = \mu_0 \vec{H} = \frac{\mu_0 I}{2\pi\rho}\,\hat{a}_\phi$$

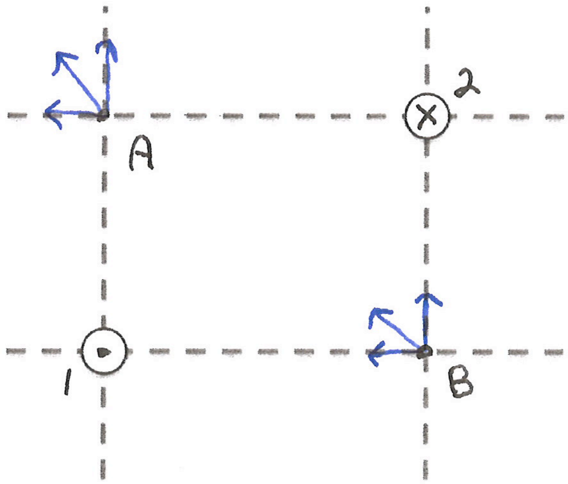
$$\vec{B}(\rho = 1m) = \frac{(4\pi\times10^{-7}\frac{H}{m})(1A)}{2\pi(1m)}\,\hat{a}_\phi = 2\times10^{-7}\frac{Wb}{m^2}\,\hat{a}_\phi$$

$$\vec{F} = +\int_0^{1m} (1A)\, dz\, \hat{a}_z \times \left(2\times10^{-7}\frac{Wb}{m^2}\,\hat{a}_\phi\right)$$

$$= 2\times10^{-7}\frac{A\,Wb}{m^2}\int_0^{1m} dz\,(-\hat{a}_\rho) = -2\times10^{-7}\frac{A\,Wb}{m}\,\hat{a}_\rho$$

$$\vec{F} = -2\times10^{-7}\,N\,\hat{a}_\rho$$

(5 pts) 9. In the diagram, wire one carries a current I flowing out-of-the page and wire 2 carries the same current I but flowing into the page.



The directions of the magnetic field intensities at positions A and B, defined at the point of intersection of the dashed lines, are,

A) ↓ ↓

B) ↑ ↓

C) ↓ ↑

D) → ←

E) ↗ ↗

F) ↘ ↘

G) ↖ ↖

H) ↘ ↖

I) ↖ ↘

J) ↑ ↑

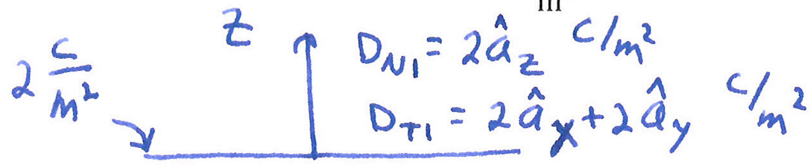
K) ↗ ↙

L) ↙ ↗

M) ↙ ↙

N) ← →

(5 pts) 10. For $z > 0$ $\mathbf{D} = 2\hat{\mathbf{a}}_x + 2\hat{\mathbf{a}}_y + 2\hat{\mathbf{a}}_z \frac{C}{m^2}$ and the relative permittivity is $\varepsilon_r = 2$. For $z<0$ the relative permittivity is $\varepsilon_r = 4$. If the $z = 0$ plane is a sheet charge of density $2 \frac{C}{m^2}$, the electric flux density for $z < 0$ is



A) $\mathbf{D} = 4\hat{\mathbf{a}}_x + 4\hat{\mathbf{a}}_y + 2\hat{\mathbf{a}}_z \frac{C}{m^2}$

B) $\mathbf{D} = 4\hat{\mathbf{a}}_x + 4\hat{\mathbf{a}}_y \frac{C}{m^2}$

C) $\mathbf{D} = 1\hat{\mathbf{a}}_x + 1\hat{\mathbf{a}}_y \frac{C}{m^2}$

D) $\mathbf{D} = 1\hat{\mathbf{a}}_x + 1\hat{\mathbf{a}}_y + 2\hat{\mathbf{a}}_z \frac{C}{m^2}$

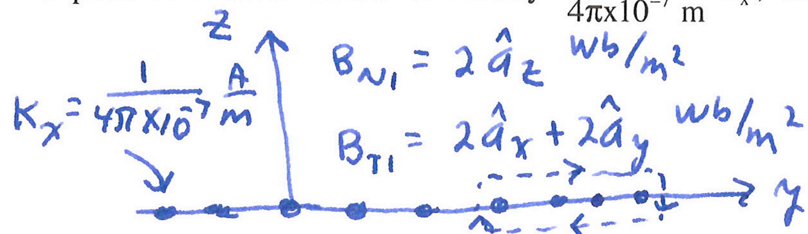
E) none of the above

$$E_{T2} = E_{T1} = \frac{D_{T1}}{2\epsilon_0} = \frac{1}{\epsilon_0}\hat{a}_x + \frac{1}{\epsilon_0}\hat{a}_y \; \frac{C}{m^2}$$

$$D_{T2} = 4\epsilon_0 E_{T2} = 4\hat{a}_x + 4\hat{a}_y \; \frac{C}{m^2}$$

$$D_{N1} - D_{N2} = 2\frac{C}{m^2} \qquad D_{N2} = D_{N1} - 2\frac{C}{m^2} = 0$$

(5 pts) 11. For $z > 0$ $\mathbf{B} = 2\hat{\mathbf{a}}_x + 2\hat{\mathbf{a}}_y + 2\hat{\mathbf{a}}_z \frac{Wb}{m^2}$ and the relative permeability is $\mu_r = 2$. For $z<0$ the relative permeability is $\mu_r = 1$. If the $z = 0$ plane is a sheet current of density $\frac{1}{4\pi x 10^{-7}} \frac{A}{m} \hat{a}_x$, the magnetic flux density for $z < 0$ is



A) $\mathbf{B} = 1\hat{\mathbf{a}}_x + 2\hat{\mathbf{a}}_y + 2\hat{\mathbf{a}}_z \frac{Wb}{m^2}$

B) $\mathbf{B} = 2\hat{\mathbf{a}}_x + 1\hat{\mathbf{a}}_y + 2\hat{\mathbf{a}}_z \frac{Wb}{m^2}$

C) $\mathbf{B} = 4\hat{\mathbf{a}}_x + 2\hat{\mathbf{a}}_y + 2\hat{\mathbf{a}}_z \frac{Wb}{m^2}$

D) $\mathbf{B} = 2\hat{\mathbf{a}}_x + 4\hat{\mathbf{a}}_y + 2\hat{\mathbf{a}}_z \frac{Wb}{m^2}$

E) none of the above

$$B_{N2} = B_{N1} = 2\hat{a}_z \; \frac{Wb}{m^2}$$

$$H_{2x} = H_{1x} = \frac{B_{1x}}{2\mu_0} = \frac{2}{2\mu_0} = \frac{1}{\mu_0}$$

$$B_{2x} = \mu_0 H_{2x} = 1$$

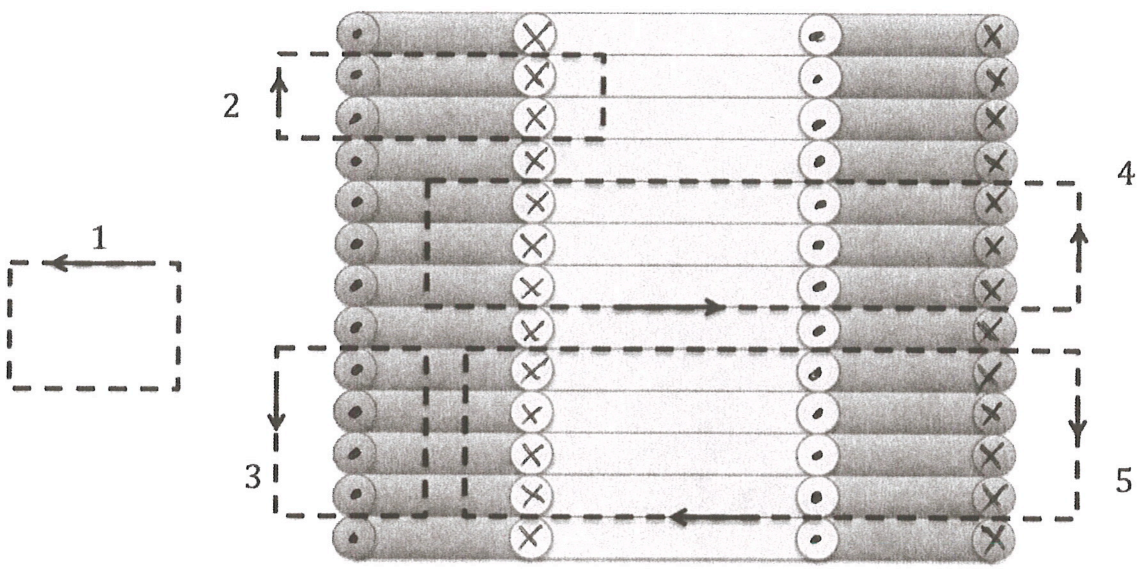
$$H_{1y} W - H_{2y} W = \frac{-1}{4\pi \times 10^{-7}} W$$

$$H_{2y} = H_{1y} + \frac{1}{4\pi \times 10^{-7}} = \frac{2}{2\mu_0} + \frac{1}{4\pi \times 10^{-7}} = \frac{2}{\mu_0}$$

$$B_{2y} = \mu_0 H_{2y} = 2$$

$$\vec{B}_2 = 1\hat{a}_x + 2\hat{a}_y + 2\hat{a}_z \; \frac{Wb}{m^2}$$

(10 pts) 12. Two solenoids have the same number of turns per unit length. They have different diameters and are co-axial. Shown is a cut through the length of the two solenoids indicating the direction of current flow where the same current, 1 A, is flowing in each solenoid. For the paths shown, which are in the same plane as the cut shown through the solenoid, evaluate the following integrals,



$\oint_1 \mathbf{H} \bullet \mathbf{dl} =$ 0

$\oint_2 \mathbf{H} \bullet \mathbf{dl} =$ 0

$\oint_3 \mathbf{H} \bullet \mathbf{dl} =$ 4A

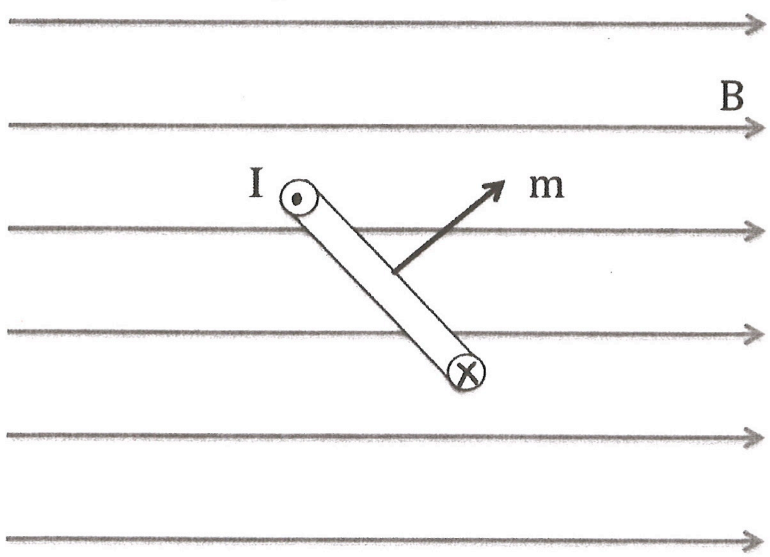
$\oint_4 \mathbf{H} \bullet \mathbf{dl} =$ -3A

$\oint_5 \mathbf{H} \bullet \mathbf{dl} =$ 4A

(6 pts) 13. Fill in the table with the standard units for the following

magnetic flux density, **B**	$\frac{Wb}{m^2}$ or T
Magnetic field intensity, **H**	A/m
Electric Field Intensity, **E**	V/m
Electric Flux Density, **D**	C/m^2
Electric flux, **Ψ**	C
Magnetic flux, **Ψ**	Wb

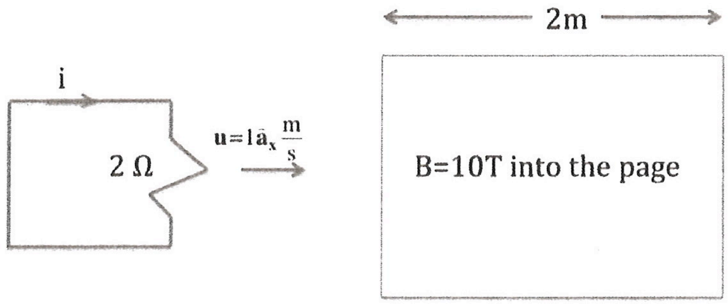
(5 pts) 14. A circular loop of wire carrying current I is in a constant magnetic flux density as show. Shown is a cut through the circular lool.

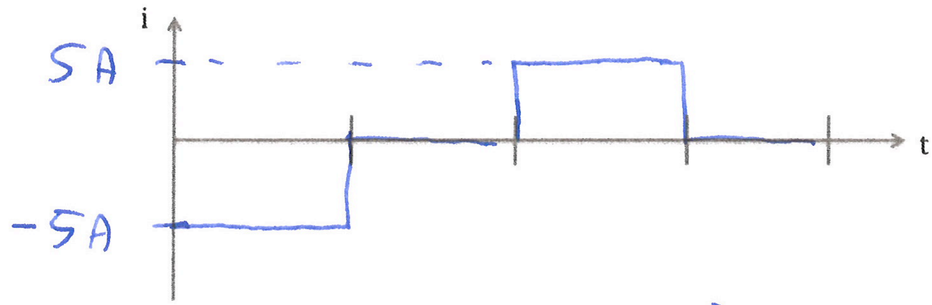


The circular loop will

A) will not rotate because the net magnetic force on a current loop in a constant magnetic field is zero.
B) rotate clockwise
C) rotate counterclockwise

(15 pts) 15. A 1m x 1m square loop contains a 2 Ω resistor and is moving at 1 m/s towards a region of uniform magnetic field 10 T into the page and of width 2 m. Assume the rectangular loop begins to enter the 2 m wide region of uniform magnetic field at t=0. Plot the current versus time in the 2 Ω resistor from t = 0 to t = 7 s. Assume the current is positive if flowing in a clockwise direction.





$0 < t < 1s$ the loop is entering the $\vec{B}$-field

A = area in the $\vec{B}$-field $= ut$

$\psi = ut\left(10\frac{Wb}{m^2}\right) \qquad \frac{d\psi}{dt} = u\left(10\frac{Wb}{m^2}\right) = 1\frac{m}{s}\left(10\frac{Wb}{m^2}\right) = 10V$

i will flow in a direction to oppose this increase in flux into the loop so $i = -\frac{10V}{2\Omega} = -5A$

$1s < t < 2s$ loop is completely in the B-field. so $d\psi/dt = 0 \Rightarrow i = 0$

$2s < t < 3s$ the loop is leaving the B-field. The flux into the loop will now decrease at the same rate it was increasing for $0 < t < 1s$ so $i = +5A$

$t > 3s$ the loop is out of the B-field so $\psi = 0$, $d\psi/dt = 0$, and $i = 0$

(16 pts) 16. The following is the equation of the magnetic field intensity of an EM wave in free space.

$$\mathbf{H} = 10\cos(1.884\text{x}10^{7}t - 6.28\text{x}10^{-2}z)\hat{\mathbf{a}}_x \frac{\text{A}}{\text{m}}$$

A) What is the wavelength of the wave?

$$\beta = \frac{2\pi}{\lambda} = 6.28 \times 10^{-2}\ m^{-1}$$

$$\lambda = 100\ m$$

B) What is the frequency of the wave in Hz?

$$\omega = 2\pi f = 1.884 \times 10^{7}\ s^{-1}$$

$$f = 3 \times 10^{6}\ Hz$$

C) What is the velocity of the wave?

$$u = \frac{\omega}{\beta} = \frac{1.884 \times 10^{7}\ s^{-1}}{6.28 \times 10^{-2}\ m^{-1}} = 3 \times 10^{8}\ \frac{m}{s}$$

$$\vec{u} = 3 \times 10^{8}\ \frac{m}{s}\ \hat{a}_z$$

D) What is the electric field intensity of the wave?

$$|\vec{E}| = \eta_o |\vec{H}| = (377\ \Omega)(10\tfrac{A}{m}) = 3770\ \frac{V}{m}$$

$$\vec{E} = -3770 \cos(1.884 \times 10^{7} t - 6.28 \times 10^{-2} z)\ \hat{a}_y\ \frac{V}{m}$$

(20 pts) 17. A plane wave that has $\mathbf{E_i} = 50\sin(6\text{x}10^8\pi t - 4\pi y)\hat{a}_x \frac{\text{V}}{\text{m}}$ travels in a lossless dielectric with $\varepsilon_1 = 4\varepsilon_o$, $\mu_1 = \mu_o$, and impinges normally onto a lossless dielectric with $\varepsilon_2 = 16\varepsilon_o$ and $\mu_2 = \mu_o$. Determine the complete expressions for $\mathbf{E_r}$, $\mathbf{H_r}$, $\mathbf{E_t}$, and $\mathbf{H_t}$.

$$\eta_1 = \sqrt{\frac{\mu_o}{4\epsilon_o}} = \sqrt{\frac{12.6\times10^{-7}\ H/m}{4(8.854\times10^{-12}\ F/m}} \approx 188.6\ \Omega$$

$$\eta_2 = \sqrt{\frac{\mu_o}{16\epsilon_o}} = \sqrt{\frac{12.6\times10^{-7}\ H/m}{16(8.854\times10^{-12}\ F/m)}} = 94.31\ \Omega$$

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{94.31 - 188.6}{94.31 + 188.6} = -0.333$$

$$\tau = \frac{2\eta_2}{\eta_2 + \eta_1} = \frac{2(94.31)}{94.31 + 188.6} = +0.667$$

$$u_2 = \frac{1}{\sqrt{\mu_o 16\epsilon_o}} = \frac{1}{\sqrt{(12.6\times10^{-7}\frac{H}{m})(16)(8.854\times10^{-12}\frac{F}{m})}} = 7.5\times10^{7}\ \frac{m}{s}$$

$$\beta_2 = \frac{\omega}{u_2} = \frac{6\times10^8\pi\ s^{-1}}{7.5\times10^7\ m/s} = 8\pi\ m^{-1}$$

$$\vec{E}_r = -16.67\sin(6\times10^8\pi t + 4\pi y)\,\hat{a}_x\ \frac{V}{m}$$

$$\vec{H}_r = -\frac{16.67}{188.6}\sin(6\times10^8\pi t + 4\pi y)\,\hat{a}_z\ \frac{A}{m}$$

$$= -0.08837\sin(6\times10^8\pi t + 4\pi y)\,\hat{a}_z\ \frac{A}{m}$$

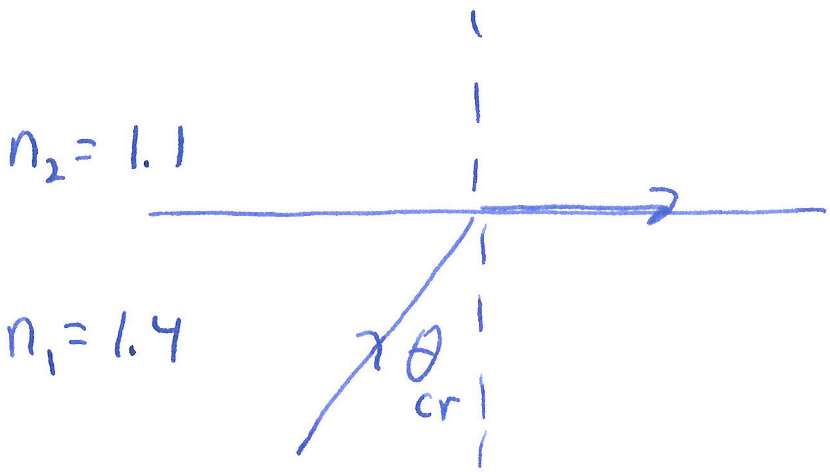
$$\vec{E}_t = 33.35\sin(6\times10^8\pi t - 8\pi y)\,\hat{a}_x\ \frac{V}{m}$$

$$\vec{H}_t = -\frac{33.35}{94.31}\sin(6\times10^8\pi t - 8\pi y)\,\hat{a}_z\ \frac{A}{m}$$

$$= -0.354\sin(6\times10^8\pi t - 8\pi y)\,\hat{a}_z\ \frac{A}{m}$$

(6 pts) 18. Material A has an index of refraction of 1.4 and material B an index of refraction of 1.1. For light in material A, the critical angle at the boundary between the two materials is

A) 0°
B) 38.2°
C) 51.8°
D) 90°
E) 180°
F) none of the above



$$n_1 \sin\theta_{cr} = n_2 \sin\frac{\pi}{2} = n_2$$

$$\sin\theta_{cr} = \frac{n_2}{n_1} = \frac{1.1}{1.4} = 0.7857$$

$$\theta_{cr} = \sin^{-1}(0.7857) = 51.78°$$